Seabed Variability and its Inluence on Acoustic Prediction Uncertainty

Model and Data Variance and Resolution: How Do We Quantify Uncertainty?

Robert I. Odom
Applied Physics Laboratory
College of Ocean and Fisheries Sciences
University of Washington
1013 N.E. 40th Street
phone: (206)685-3788 fax: (206)543-6785 email: odom@apl.washington.edu

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LONG-TERM GOALS

A basic tenet of the Office of Naval Research's Uncertainty DRI is that, in any strategic situation, environmental parameters will never be known in complete enough detail to enable a perfectly accurate acoustic detection. In order to address the problem of unknown uncertainty this research is focused on two goals: 1. Assess and characterize seafloor variability in shelf areas. 2. Determine the impact of the seafloor variability on acoustic prediction uncertainty.

OBJECTIVES

The primary focus of this project will be to compute model and data sensitivities, investigate model parameter correlations, and optimal model parameterizations. The objectives can be stated as the answers to the following questions: Given the data we have available or can reasonably expect to be able to collect, which model parameters are most important? Can we resolve them? What will be the model variance? What additional data would be useful, if it were available? Which model parameters are essentially unresolvable (unconstrained)?

APPROACH

All our representations of the ocean/seafloor environment are, whether they are entries in a database or parameters supplied to a synthetic model, are under-parameterized versions of the true environment. Inverse problems that solve for environmental parameters are generally severely under-determined. This is because we attempt to represent what amount to continuous functions, e.g. water sound speed and bottom structure, with a finite number of discrete parameters. We hope that the parameters we choose capture most of the important features of the environment, but we are limited in the data we can collect by both practical considerations and physical constraints. Practically we can only collect a limited amount of data because of cost. Physical constraints make it impossible to obtain the kind of coverage available in a medical tomographic scan, for example. We simply do not have 360° coverage of our medium. Because of our limited data collecting capabilities and our finite model parameterization, we are forced to make trade-offs between variance reduction in our parameterized model and the resolution of our model (Menke, 1989).

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The issue is that our model has some variance; this is the model uncertainty, which we wish to reduce. We collect acoustic or other data, which has its own variances, and combine this data in an inverse problem to reduce the variance in whatever starting model we have chosen to represent the environment. A common approach is to search the model space for a set of parameters, which produce a model that fits the data according to some criterion like aχ^2 statistic. Obtaining a model that fits the data is only half the problem. We still need to understand the variance and also the resolution of the model we have found. In fact the commonly used χ^2 statistic tells us nothing about the model variance, because it does not take the data variances into account. A low value of χ^2 tells us we have a model that fits the data, but it does not tell us how much of what we fit might be noise.

In addition to the model and data variance, we also need to know the resolution of both our data and our model. The variance and resolution are closely connected and we ultimately have to live with some compromise between the variance reduction in our model parameters and their resolution. We have a finite amount of data, some of which may be redundant. We can parameterize our environmental very finely, in which case we must use our limited data set to determine the values of many parameters. The result will be a model with many parameters, but with relatively large variances. On the other hand we could decide to parameterize our model very coarsely, and determine only a few parameters, with a consequent loss of resolution. However, because we have expended our data estimating only a few parameters, the variances of those parameters will be relatively lower. The choice of how to invest our data, whether to reduce the variance of a few parameters or to have a more finely parameterized model with larger variances, is ours to make.

It is possible to pre-compute measures of the model and data variance and resolutions. This allows us to determine the trade-offs available between the variance and resolution. The model and data variance and resolution matrices depend on partial derivatives of the pressure with respect to density and bulk modulus ($\partial p/\partial \rho$ and $\partial p/\partial \kappa$) (Tarantola, 1984). These derivatives, referred to as functional or Frechet derivatives in the literature, can be computed either by numerical differencing, a numerically intensive procedure, or by evaluation of very convenient analytical expressions (Pan, Phinney and Odom, 1986). These derivatives quantify the sensitivity of the model to perturbations in bulk modulus and density as a function of position. The two derivatives above are the most important. Other derivatives of interest can generally be constructed by application of the chain rule for differentiation. For example if we are interested in the sensitivity of the complex pressure field to perturbations in attenuation we can obtain it from $\partial p/\partial \kappa$ by making κ complex and then differentiating κ with respect to its imaginary part α.

In addition to quantifying the model and data variances and resolutions, it is also important to understand correlations between model parameters. For example, sound speed and layer thickness are correlated (Schmidt and Baggeroer, 1995). This directly affects how we should parameterize our model. Bube et al. (1995) have provided quantitative guidelines for how to discretize the model to compute inverse solutions which are as accurate as possible in the features of the model which are well determined (resolved) by travel time data. In particular the sound speed model should not be discretized much coarsely than the reflectors as a way of stabilizing the inverse problem, because that may force the computed layer depths to try to match aspects of the data, which are caused by features in the sound speed field.

WORK COMPLETED

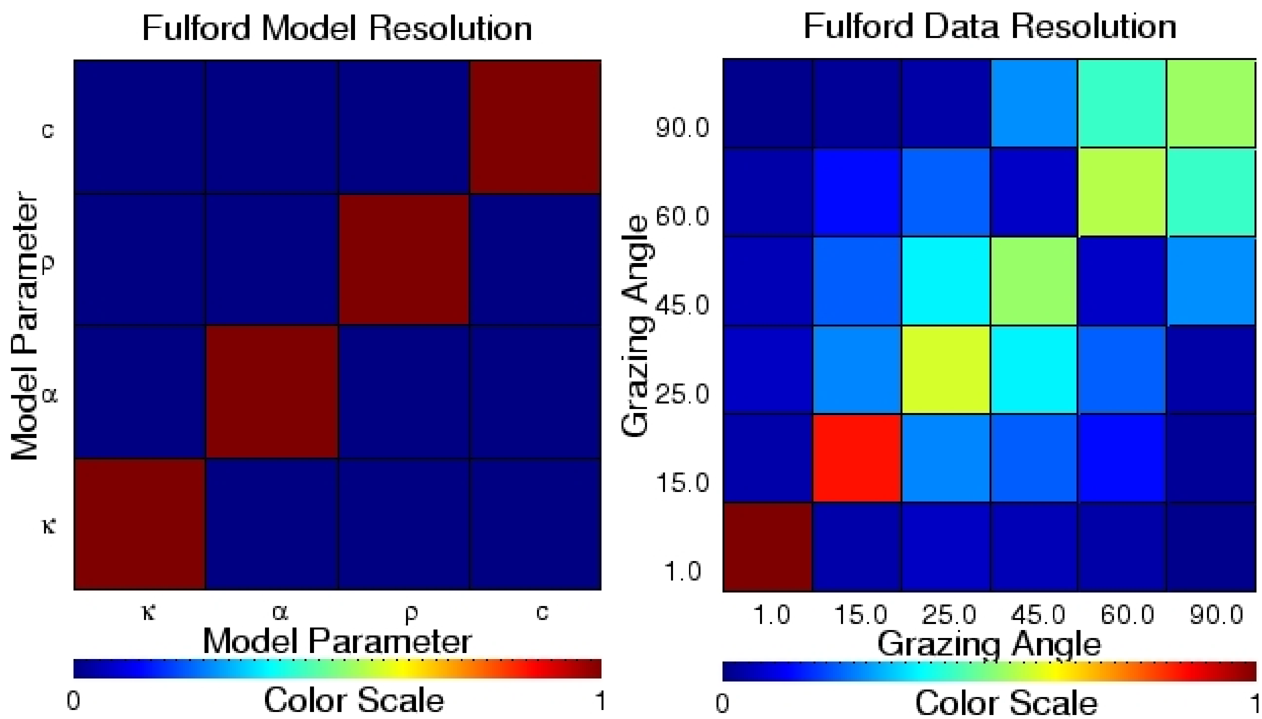
The focus of this year's effort has been on the construction of the model and data resolution matrices, **R** and **N**, respectively, and their interpretation as measures of the model uniqueness and parameter coupling. The model resolution matrix $\mathbf{R} = \mathbf{G}^{-g}\mathbf{G}$ where **G** is the N×M matrix of Frechet derivatives. The data resolution matrix $\mathbf{N} = \mathbf{G}\mathbf{G}^{-g}$. The N×N data resolution matrix **N** characterizes whether data can be independently predicted or resolved. The M×M model resolution matrix is useful for predicting to what scale features can actually be resolved and for exposing parameter couplings, and the relative magnitude of the coupling. In addition, the resolution matrices can be used together with variance estimates for trade-off studies between variance and resolution. In any inverse problem we are always faced with the issue on how to use our data. At the two extremes, we can use all of our data to estimate a single average value for one model parameter, sound speed, say. This one number will have a low variance, if a large amount of data went into its estimation. However its resolution will also be low. We have averaged over a whole region or temporal period, say. Alternatively we could use the data to estimate the sound speed at many spatial locations or many times. Our estimates will have very fine resolution, but if very little data has been used to estimate each value, those values will necessarily have large variances. The experimenter must decide how best to use the data, which will likely somewhere in between the two extremes just described. The next section illustrates some resolution matrices and their interpretation.

RESULTS

Results are illustrated in Figures 1a and 1b. Figure 1a shows a model resolution matrix for four model parameters at the water-sediment interface, assuming measurements of complex pressure at six grazing angles between 1° and 90° are available. The sediment model (not shown) is for the continental shelf area of the East China Sea, and was obtained from James Fulford of NRL-Stennis. The figure shows that the four model parameters, sound speed (*c)*, density (ρ), bulk modulus (κ) and attenuation (α) are well resolved at the water-sediment interface employing data from the six grazing angles.

Figure 1b shows the data resolution for the same sediment model. The most important feature is that the off-diagonal terms increase for higher grazing angles. That is, the ability to predict the data at higher grazing angles degrades. It becomes difficult to independently predict the data. The conclusion to be drawn is that we can find a model, which fits the data, but there are many models, which fit the data. Our model will be non-unique.

The Frechet derivatives characterize the parameter sensitivities, and can be represented as products of the Green's function for the medium and its vertical derivatives. This permits very efficient computation of the derivatives. Only one pass of a forward modeling program is required for the computation. Any program which computes the Green's function or a suitable approximation to it will suffice. The model and data resolution matrices are built up from the derivatives and contain information about parameter resolvability, parameter coupling and data independence. Their computation requires no actual data for input. They can be an aid to experimental design. It is important to remember however, that they are local in nature. If the mapping between the data and the model parameters is highly nonlinear, and contains many local minima, then some sort of global optimization should be carried out. Once a suitable global minimum in cost function has been found, a local resolution analysis can be carried out to characterize the nature of the optimal solution.



1a ***1b***

Figure 1. 1a(left) shows the model resolution matrix at the water-sediment interface for a sediment model characteristic of the East China Sea. Assuming six values of the complex pressure at six grazing angles from 1° to 90° are available, the four model parameters sound speed (c), density (ρ), bulk modulus (κ) and attenuation (α) are well resolved at the water-sediment interface. 1b(right) shows the data resolution matrix for the same sediment model and numerical experiment. The large off-diagonal components for the larger grazing angles indicate an inability to independently predict the data. That is, we can find a model, which fits the data, but the model will not be unique.

IMPACT/APPLICATIONS

Answering the questions posed in the Objectives section of this report will provide quantitative bounds on what can be expected from an optimal experiment designed for environmental characterization, how much we are giving up for a non-optimal experiment, and which environmental parameters are best and least determined and determinable.

TRANSITIONS

In the short term, the results of this research will be utilized by the other members of the Seabed Variability Team and the 6.2 Capturing Uncertainty team. In the longer term, the results of this research will permit the quantification of the effects of sampling density, scale variability, and parameter sensitivity for inclusion in seabed environmental databases.

RELATED PROJECTS

This research is directly related to the other sub-projects in the “Seabed Variability and its Influence on Acoustic Prediction Uncertainty” group.

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